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Geometry in structural mechanics education revisited

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ABSTRACT: It is widely known that geometry was created and developed to solve engineers' and other scientists' practical problems. The need for knowledge of geometry is further supported nowadays; the development of new scientific tools for structural analysis and the design of constructions require the knowledge of practical and classical geometry, as well as a wider knowledge of modern geometries. The extent of geometry education was decreased significantly from the beginning of the 20th Century, something evident at both national and international levels. The results of downgrading geometry education, in particular for engineers, have become more visible than in previous years. Moreover, as recently evidenced from national and international organisations, the qualitative upgrading of engineers is related to the quantitative and qualitative increase of geometry training in every level of education.

INTRODUCTION

The first references to geometry are attributed to the 20th Century BC in Mesopotamia and Egypt [1]. The history of Geometry in structural mechanics goes back to the construction of the pyramids [2]; it was created and developed to meet practical problems, mostly of engineers and scientists of the time. Geometry is used not only to solve practical problems, but also for the development and research in structural engineering and other sciences [3].

The need for knowledge of geometry for structural engineers has been recognised by many researchers over time, even from the early years (Figure 1) [4-7]. The real or mental conception of space, the visualisation of many mathematical concepts and shapes, the first realisation and preparation of the model simulation and other skills that are analysed in this work, can be induced and developed only through the teaching of geometry, especially, in primary and secondary education. As also discussed in this article, the contribution of classical geometry, which is taught in secondary education (SE) is also very important for structural engineers, since, for example, enhances the ability of proving procedures in solving problems.

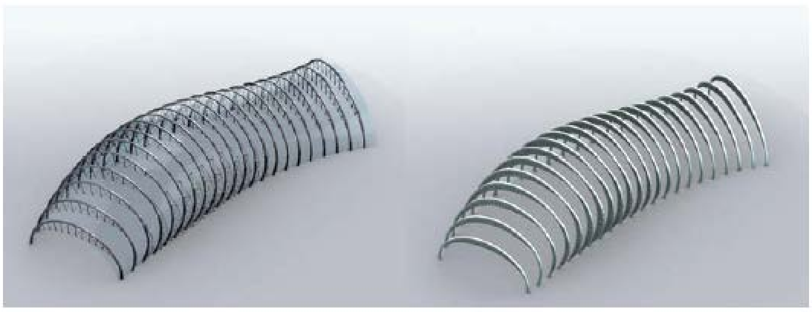

Figure 1: The structural elements of the figure were designed in a way that their geometry includes all the structural restrictions [6].

As far as higher education is concerned, analytical geometry, differential geometry, computational and non-Euclidean geometry are essential for the structural engineers as also presented in a recent research [8]. Taking into consideration the development of new scientific tools (e.g. finite elements), the need for knowledge of geometry became even more necessary for structural engineers. The contribution of geometry to research and development of structural mechanics is also a subject of this article.

In recent years, geometry education has decreased in the full range of engineering education in primary, secondary [9][10] and tertiary education in Greece [8], and internationally [2][8][11][12]. This statement is also supported by recent studies [12][13], in which the need for geometry education has been emphasised. These studies resulted in: a) the lack of quality engineers in the United Kingdom; and b) the need to import engineers from other countries. Both results are due to inadequate teaching of mathematics and the reduction of their teaching in secondary and undergraduate level education [12]. Among them (mathematics) geometry is a significant part of engineers' education. The lack of knowledge of geometry, among other factors referred in this work, decreases the knowledge potential of structural engineers, since most of the concepts are geometric principles or/and use a wide range of geometry [2].

All the above mentioned consequences ought to be reversed. The reverse procedure may include several steps and different measures. At first, appropriate tools (e.g. e-material) at the school level (primary and secondary education) should be used to increase interest of the educated groups. In the higher education system, mathematics curricula should be reconsidered in the sense of increasing quality and quantity of geometry education; the same consideration applies for engineering curricula.

This article is organised as it follows. A description on the importance of geometry in structural engineering education is initially presented, with respect to all levels of education and emphasis in structural engineering departments. It turns out that the lack of adequate teaching of geometry at all levels of education leads to quality degradation of structural engineers. The need for geometry knowledge is analysed and the effects of the lack of geometry knowledge are presented in a later subsection. Finally, conclusions and suggestions to improve engineering education with respect to the increase of geometry education are presented in the last section.

GEOMETRY AND SECONDARY EDUCATION

According to several authors [9][10][14], the pedagogical value of geometry is indisputable, since: a) it helps in the ability of space perception; b) enhances the ability of space mental perception; c) connects mathematics to the real world; d) it helps in understanding of abstract mathematical ideas from other areas of mathematics, through the interpretation of geometric models; e) it is a unique basis for the rational use of proof logic in all practical applications; f) it is an excellent example of a complete mathematical system, in fact, it is the most simple and understandable for students and pupils; and g) it promotes imagination, creativity, spatial perception, complex thinking; in particular, for the SE, it helps on the perception of dimensional space and superimposition principles.

Geometry education starts in the primary school where originally practical geometry is taught; many of the above mentioned benefits are achieved by practical geometry. Practical geometry might be considered as the first important step for the education of the structural engineer. Practical geometry is the only way (at this level of education) for students to visualise, not only, geometric concepts and shapes, but more importantly, the majority of the mathematical content taught in primary and secondary as most of these can be taught and demonstrated by using geometry.

The next step of geometry education is classical geometry. In many countries the teaching of classical geometry (CG) is quite degraded [12] and, in particular, the part relating to the three-dimensional geometry. This choice is in contrast to the statement made by some researchers [9], which highlight the very important contribution of CG in the mental development of children through learning strategies to solve problems so as to enhance the logical, creative and critical thinking. Teaching classical geometry in the high school can also help students to gain the sense of building mathematical theories, the concept of proof in mathematics and develop skills to use proofing processes in solving problems. The concepts described in the classical geometry help students to recognise the role of shape in geometry as a component directly related to the geometric thinking. It should be also noted that the concepts of symmetry and proportionality in structural engineering are first described and perceived by classical geometry. Finally, one should mention that classical geometry is the transition from empirical to theoretical thinking.

GEOMETRY AND STRUCTURAL ENGINEERING EDUCATION

It is well known that mathematics is the most important tool for engineers [15] and, this view is supported by recent studies; a very important part of mathematics is geometry [8][12][13]. Geometry has an important role in the design [14] and the construction of structural elements, as well as of the whole construction, e.g. it influences the distribution of the applied load in the structure i.e. different shapes of the structure leads to different internal forces [16]. Carpinteri also mentions the influence of geometry on the strength of materials [17].

Kent and Noss stress the importance of knowledge of geometry for site and industry engineers [18]. They present research where most site engineers believe that the use of the knowledge of geometry and, in general, mathematics in production is not very important, except perhaps their use in the design of structures. In contrast, Kent and Noss demonstrate the importance of geometry, not only for spatial perception and construction, but also for the understanding and awareness of the structural behaviour of a construction. Examples of such behaviour are the bending of a beam, which is a parabola, the structural behaviour of an H section and the balance of power in three-dimensional tents. Furthermore, the *structural feel* [18] is very useful in both the design and construction for the structural engineer; part of this ability is acquired by geometry education, i.e. some authors comment [19] that catastrophic failures in constructions are due, among other things, to the wrong conception of the structural geometry (an important example can be considered the geometric regularity of a building defined in Greek standards).

The need of geometry for the structural engineer is further supported in recent works [4-6]. The authors stress the need for three-dimensional visualisation of constructions' structural design. This need is further enhanced considering the development of new scientific tools (e.g. finite elements) and the new structural elements (shells, membranes) used in the design and the construction (Figure 2) [5][14][20][21]. Moreover, the knowledge of non-Euclidean geometries are also important for the structural engineer, especially for the design and analysis of modern structural elements as, for example, the church temples, which are using spherical triangles. In addition, other researchers also report that the lack of knowledge of geometry makes it more difficult to understand most concepts of structural mechanics since, as is clear from contemporary reports, much of this is based on geometric principles or uses a wide range of geometry [2], e.g. the geometric definition of stresses, geometric stability of structures [2] and the principle of virtual work [7].

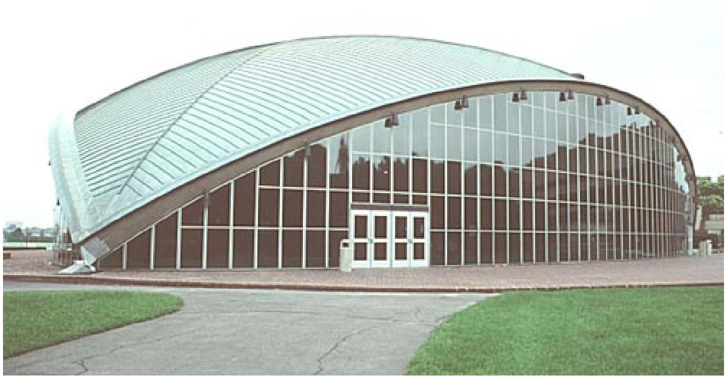

Figure 2: Roof shell constructed using reinforced concrete, Kresge Auditorium, MIT campus, USA [21].

In research concerning the need of geometry in theoretical and applied mechanics (and subsequently in structural engineering), Liapi [14] states the importance of Euclidean geometry in mechanics and, in particular, connects the conditions of rigidity of Euclidean solids studied by Cauchy [22] with the behaviour of structural construction's elements [23-25], i.e. the rigidity of a construction depends on the geometry of the structural elements that define the construction. An example that confirms the above is the following: the compound single bonds (joints) of the rigid plates in a convex polyhedron is a rigid construction, where in a non-convex polyhedron, the structure can be rigid, be infinitesimally moveable, have multiple equilibria or be mechanism [22][24].

Furthermore, the concept of geometric variability in continuum mechanics (important background for the structural engineer) is also very important. It was developed by Euler, Lagrange and Hamilton and requires special knowledge of Euclidean and non-Euclidean geometry. One such example is the use of variable integration methods in classical engineering theory (Lagrange, Hamilton) based on geometry that takes into account all the symmetries of space to delimit invariant integration quantities [26]. Additionally, Mora emphasises the use of geometric limits in the analysis and design of structures [27]; Niemeier indicates the usefulness of geometry in more automated manufacturing construction processes [28]; Laschauer and Kotnink introduce geometric methods to introduce structural constraints in the design of structures [6]; Schmidt et al stress the direct dependence of geometric design and structural analysis of constructions [29]. Finally, Barthelemy and Haftka [30] and Kirch also emphasise on the influence of geometry in optimal structural design of constructions [31][32].

Another aspect that supports the importance of geometry in structural mechanics is computational geometry, a recent scientific field, which was created to meet mostly the needs of engineers considering the development of new tools in the design and analysis of structures, as already mentioned above [5]. Research in computational geometry includes, among other topics, the investigation of geometry influence on the structural characteristics of data structures; see also (http://structuralmorphology.org/). Furthermore, the influence of geometry in mechanics has been also recently recognised by the American Institute of Mathematical Sciences, which announced in 2009 a specialised scientific publication *The Journal of Geometric Mechanics* (JGM). JGM publishes applications of geometry in engineering with reference to all sectors underlying structural mechanics (continuum mechanics, statics, dynamics, mechanics of solids, etc).

Finally, it is important to notice the bidirectional connection of Euclidean geometry and classical mechanics (background of structural mechanics), i.e. the concept of Euclidean geometry can be used as basis for creation and development of classical mechanics and *vice versa* [26][33], i.e. it is possible to prove geometrical concepts, e.g. the centre of gravity of a triangle using an engineering approach [33]. In addition to the previous statement, several researchers report that geometry can be taught using the concept of structural stability [34][35], e.g. dynamic software can be used for the proof (based on structural stability) of geometry theorems, such as the definition of a plane using a straight line and a point.

QUANTITATIVE AND QUALITATIVE EVALUATION OF TEACHING GEOMETRY IN EDUCATION

It is important to mention that in previous centuries, mathematicians were called geometers due to the large proportion of geometry in mathematics courses in the early years [36]. The reduction of geometry education started from the beginning of the 20th Century [42][45], e.g. geometry in Greek primary education does not exceed 16% of the hours devoted to mathematics [9] and in secondary education the decrease of teaching of 3D Euclidean geometry [37] started the decade 1990-2000.

A decrease of teaching in geometry has also occurred in Greek tertiary education [2][8][9] and internationally [39]. According to Liapi, teaching descriptive and analytical geometry has been recently reduced in the US [14]. On the contrary, this has not been observed in most of the Greek structural education departments. The same has happened with the teaching of analytical geometry in mathematical schools. However, the lack of teaching analytical and differential geometry in some structural education departments has been pointed out in a study by the Technical Chamber of Greece [8]. Note also that teaching of non-Euclidean geometry is not included in all the mathematics departments of Greece.

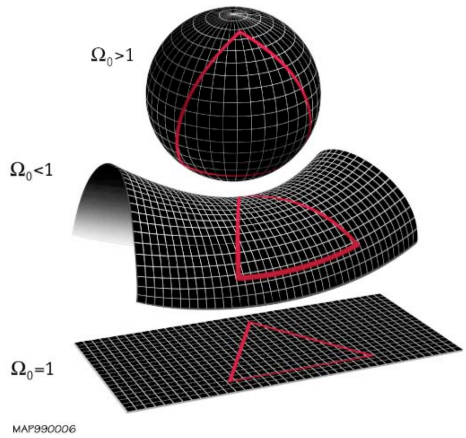

Figure 3: Euclidean ($\Omega_0 = 1$) and non-Euclidean geometries ($\Omega_0 < 1$ - hyperbolic geometry, $\Omega_0 > 1$ - spherical geometry) [38].

RESULTS OF LACK OF GEOMETRY KNOWLEDGE IN STRUCTURAL ENGINEERS

Most of the consequences induced from the downgrading of geometry education can be extracted from the absence of the advantages of geometry education discussed above. The geometric perception, intuition and imagination have been significantly reduced in the new generations as has been confirmed by several researchers [15]. The lack and/or degradation of teaching classical geometry in high school, downgrades the cognitive development of children as already explained in previous paragraphs. Furthermore, very important concepts as is symmetry and proportion, which are very well established and defined using geometric terms, become more difficult to be taught; this statement is even more strengthened for 3D geometry. Regarding higher education, the lack of teaching 2D and 3D classical geometry decreases the ability of structure visualisation, decreases structural feel, and reduces the capabilities of the engineers in the use of modern methods in the analysis and design of structures. Note that the absence of 3D Euclidean geometry creates a gap in the theoretical background needed for descriptive, analytical and differential geometry.

Finally, new research concerning engineers’ competence was recently presented by the Royal Academy of Engineering with the help of educational and production units in the UK [12]. The research shows that teaching geometry is necessary to upgrade the skills of engineers. These studies identify the lack of teaching of mathematics in technical universities and attribute it to the lack of necessary knowledge from the educators. Note that these studies were performed with the substantial contribution of large engineering firms in Europe that highlight and indicate the need for increase of core courses in engineering education [12][13][45].

CONCLUSIONS - SUGGESTIONS

Geometry education in secondary education can be improved by using digital technology and, especially, relevant dynamic software (Figure 4), e.g. Gutierrez [46]. Some studies have shown that the use of such software can even contribute to the development of students' ability to explore, to create logical statements and the ability to develop mathematical reasoning [9][41][44]; these are the reasons for which these kind of methods are recommended by the Greek secondary education curriculum [37]. However, teachers’ choices on the use of dynamic software in the classroom and the choice of appropriate mathematical activities, determine the effectiveness of these tools. Furthermore, the use of engineering proofs for geometry concepts in the teaching of geometry at all levels of education, in particular, using dynamic software, could partially reverse the previously mentioned consequences, since it would be substituting a significant part of geometry education existing before the downgrading of the 20th Century.

In addition, in order to further reverse the consequences mentioned in the previous paragraphs, the curricula of mathematics departments should be adjusted towards the improvement of geometry studies. Finally, the teaching of geometry in schools/departments dealing in engineering should be quantitatively and qualitatively improved, especially, in the sense posed by the authors of, stating that geometry can be taught using the concept of structural stability [34][35]. Especially, in the schools of construction engineering, teaching mathematics and introductory concepts of structural stability of structures can be delivered using geometric and dynamic software.

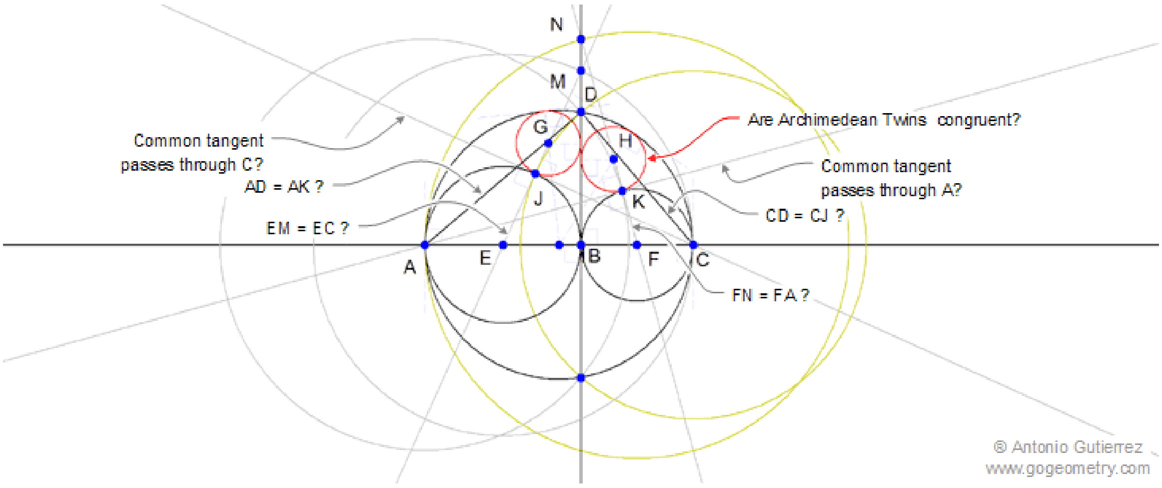

Figure 4: Visualisation of the Archimedean Twins and a variety of conjectures using Geometry Expressions® software [46].

Finally, it can be concluded as follows:

- Qualitative and quantitative teaching of geometry for structural engineering requires adjustment of curricula from secondary education.
- The direct and indirect effects of geometry education in structural mechanics are very important for all construction engineers, designers and researchers.
- Downgrading of geometry education at all levels of education started at the beginning of the 20th Century and it still exists.
- Recent studies also associate the quality of engineering with significant mathematical background, which includes geometry. The same studies suggest the replacement of specialised courses with teaching important mathematical background.
- Upgrading geometry teaching should include use of specialised dynamic software.
- Upgrading the teaching of geometry also requires training of the trainers. In this direction, adjustment of mathematics department curricula in geometry and geometry education are required.
- Finally, in particular for structural education departments, one may propose teaching of both mathematics and introductory courses in engineering to be performed using dynamic geometric software and geometrical concepts.

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